Comparison of Five Conditional Probabilities in 2-level Image Thresholding Based on Bayesian Formulation

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Abstract

In this paper, an efficient method for two-level thresholding is proposed based on the Bayes' formula and the maximum entropy principle, in which no assumptions of the image histogram are made. Five forms of conditional probability distributions, *Simple, Linear, Parabola Concave, Parabola Convex* and *S-Function*, are employed and compared to each other for optimal threshold determination. The experiment results show that the *Parabola Concave* form is the most effective, retaining most of the information for most thresholding images. The *Linear* form is an acceptable form due to its simple first-order linear function.

1. Introduction

Many thresholding methods have been developed in the last two decades. Among them, the histogram-based thresholding technique has been dominant since it only needs the gray-level histogram without other priori knowledge. While global thresholding methods, in which the entire image is thresholded with a single threshold value, have been widely applied because they are independent of the image size and also effective. In this paper, an efficient optimal thresholding method is proposed based on Bayes' formula and the maximum entropy principle. The *Linear* [1] and *S-Function* [2] forms are usually considered in fuzzy method. We here propose other two nonlinear functions, *Parabola Concave* and *Parabola Convex.* Unlike the likelihood method where the Gaussian distribution is assumed, no assumptions are made by our method.

2. Method Based on Bayes' Formula and Maximum Entropy

The aim of two-level thresholding of an image *I* is to separate its domain *D* into two parts, D_d^* and D_b^*, where D_d^* is composed of *'dark'* pixels and D_b^* is composed of those *'bright'* pixels. This classification involves a determination of the optimal threshold $\tilde{g}$ such that a pixel $I(i,j)$ is classified into D_d^*, if $I(i,j) \le \tilde{g}$; or D_b^*, if $I(i,j) > \tilde{g}$ under the condition that the information given by the original image is preserved as much as possible after this partition. However, due to the fact that the boundary between bright and dark is not well defined, some of the pixels with the same level may be classified into D_d^* and others may be classified into D_b^*. This situation must be taken into account in determining the threshold value $\tilde{g}$. It is assumed therefore that for each $g \in G$, D_g is composed of two parts, D_{dg} and D_{bg}.

Let $P_d^* = P(D_d^*)$ and $P_b^* = P(D_b^*)$. Based on the complete probability formula, we therefore have

$$P_d^* = \sum_{g=0}^{L-1} p_g \cdot p_{d|g} \quad \text{and} \quad P_b^* = \sum_{g=0}^{L-1} p_g \cdot p_{b|g} \qquad (1)$$

Note that $p_{b|g} = 1 - p_{d|g}$, therefore

$$P_b^* = 1 - P_d^* \qquad (2)$$

We adopt the Shannon maximum entropy theory as the criterion of our method. the entropy function *E* of partition is defined as

$$E = -P_d^* \lg P_d^* - P_b^* \lg P_b^* \qquad (3)$$

where P_d^* and P_b^* are given in Equation (1)**.** Using Equation (2), the optimal value of $\tilde{p}_d$ in Equation (3) can be found as :

$$\tilde{p}_d = p_d^* = \frac{1}{2} \qquad (4)$$

3. Forms of $p_{d|g}$ and $p_{b|g}$

To determine the entropy, the conditional probability functions $p_{d|g}$ and $p_{b|g}$ should have the property in equation (1). Following function form satisfies this property ($p_{b|g} = 1 - p_{d|g}$):

$$p_{d|g}(g,a,c) = \begin{cases} 1 & 0 \le g \le a \\ f(g,a,c) & a \le g \le c \\ 0 & c \le g \le L-1 \end{cases} \qquad (5)$$

We select five functions with 0th-order, first-order and second-order respectively as follows:

(a) *Simple*

this form is just with one parameter *a*. This is actually a special case of the following four forms when *a* = *c*.

(b) Linear

$$p_{d|g}(g,a,c) = \frac{g-c}{a-c} \qquad a \le g \le c$$

(c) *Parabola Convex* (with the apex in *a*):

$$p_{d|g}(g,a,c) = \frac{-g^2 + 2ag + c(c-2a)}{(a-c)^2} \qquad a \le g \le c$$

(d) *Parabola Concave* (with the apex in *c*):

$$p_{d|g}(g,a,c) = \frac{(g-c)^2}{(a-c)^2} \qquad a \le g \le c$$

(e) *S-Function*

$$p_{d|g}(g,a,c) = \begin{cases} 1 - 2\left(\frac{g-a}{c-a}\right)^2 & a \le g \le \frac{a+c}{2} \\ 2\left(\frac{g-c}{c-a}\right)^2 & \frac{a+c}{2} \le g \le c \end{cases}$$

This *S-Function* is symmetrical at the cross-over point $g = \frac{a+c}{2}$.

4. Experiment Results

The experiments with bi-level thresholding on various kinds of images have been carried out with the proposed method using proposed five forms of conditional probability distribution functions. The four original images with various histogram distributions are selected. Each image is presented by eight bits, that is, $L = 256$ from 0 (the darkest) to 255 (the brightest).

According to equation (4), we obtain the maximum entropy when $p_d^* = 0.5$. However, p_d^* is not equal to 0.5 exactly in most cases due to the discrete histogram. So we may use following minimum error *MinErr* instead of maximum entropy to find the optimal threshold $\tilde{g}$ in our experiment:

$$MinErr = \left| p_d - 0.5 \right| = \left| \sum_{g=0}^{c} p_{d|g} h_g - 0.5 \right|$$

Table 1 gives the results of optimal thresholds $\tilde{g}$, corresponding parameters $(\tilde{a}, \tilde{c})$ and the minimum errors *MinErr* using the five forms of conditional probability functions $p_{d|g}$. It is obvious that the error from the *Simple* form is much higher than that from the other four forms for all images. However for some images, the *Simple* form obtains the same optimal threshold as some of the other forms. For example, the threshold *200* is obtained using both the *Simple* and *S-Funcion* forms for the image *1*. Another example is for the image *2* in which threshold *130* is obtained from the *Simple* and *Parabola Concave* forms. So the *Simple* form can not be ignored due to its brevity.

By comparing the five forms of $p_{d|g}$ from Table 1, it can be seen that the *Linear* and *Parabola* forms (*Concave* and *Convex*) achieve a smaller error than the *S-Function* form and an even smaller error than the *Simple* form. The minimum errors for the five forms for each image are marked in bold font in Table 1. That is, the *Linear* form gives the best results for the images *1*, *Parabola Concave* gives the best results for *the image 3*, and *Parabola Convex* for the images *2* and *4*. Therefore, it appears that the *S-Function* form is not a

Table 1. Optimal thresholds and corresponding errors

Image		***Simple***	***Linear***	***Parabola Concave***	***Parabola convex***	***S-Function***
1	$\tilde{g}$	200	183	196	186	200
	$(\tilde{a}, \tilde{c})$	(200, 200)	(118, 249)	(184, 225)	(25, 252)	(137, 247)
	MinErr($x10^{-5}$)	1269.53	**1.520**	13.82	1.926	1269.5
2	$\tilde{g}$	130	131	130	133	131
	$(\tilde{a}, \tilde{c})$	(130, 130)	(84, 179)	(129, 132)	(91, 151)	(89, 174)
	MinErr($x10^{-5}$)	397.9	4.861	2.116	**0.016**	0.545
3	$\tilde{g}$	128	123	127	129	124
	$(\tilde{a}, \tilde{c})$	(128, 128)	(101, 153)	(121, 143)	(7, 180)	(9, 240)
	MinErr($x10^{-5}$)	190.7	1.996	**0.492**	1.654	0.645
4	$\tilde{g}$	146	135	136	142	138
	$(\tilde{a}, \tilde{c})$	(146, 146)	(30, 240)	(91, 244)	(20, 193)	(22, 255)
	MinErr($x10^{-5}$)	445.96	1.352	4.406	**0.624**	0.662

better choice of $p_{d|g}$.

5. Conclusions

An efficient two-level thresholding method for monochrome images is proposed in this paper. The method is derived based on Bayes' formula and the maximum entropy principle in which the entropy function is given by a conditional probability function $p_{d|g}, g \in G$ and the histogram H of the considered image. The optimal threshold $\tilde{g}$ is obtained from the conditional probability function $\tilde{p}_{d|g}$ which makes the entropy function $E(p_{d|g})$ arrive at the maximum value.

Compared with the maximum likelihood method, no assumption is made in our proposed method. To study the effect of the form of $p_{d|g}$, five forms of $p_{d|g}$ were used in the computation. The experiment results show that the *Parabola Convex* form is the most effective, retaining most of the information for most thresholding images. The *Linear* form is also an acceptable form due to its simple first-order linear function, while the *S-Function* form performs poorly and consists of two second-order nonlinear functions.

References

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