

# Reduction of Artifacts in Cosine Transform Coded Images

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## Abstract

A method for reducing blocking artifacts in compressed images coded by the block discrete cosine transform (BDCT) is presented. Problems arising from the independent transform and quantization of individual blocks result in blocking artifacts. This study is aimed at reducing the blocking artifacts while avoiding excessively blurring the underlying image. The theory of projection onto convex sets (POCS) is applied. An image before compression is simulated by a locally adaptive triangular mesh. The mesh forms the basis of a smoothness constraint set. To construct such a mesh, each image block is divided into a set of triangles. The number of triangles in a block is determined by the intensity variation of the block. Experimental results show that the proposed method outperforms two competing methods.

**Keywords:** BDCT, blocking artifact, JPEG, MPEG, POCS.

## 1 Introduction

The block discrete cosine transform (BDCT) is a widely used technique for image and video compression. It is the basis of a number of international standards, such as JPEG, MPEG, MPEG-2 and MPEG-4. The compression technique is based on dividing an image into small blocks, taking the cosine transform of each block and quantizing the transform coefficients. Since each block is transformed and quantized independently, a BDCT coded image often presents discontinuities between blocks especially at high compression ratios. Such discontinuities appear as blocking artifacts.

Blocking artifacts can be reduced by applying the theory of projection onto convex sets (POCS) [Youla and Webb, 1982]. The idea is to impose a set of constraints to an encoded image so as to make the decoded image approach its original artifact-free form. Two types of constraints, namely *quantization constraints* and *smoothness constraints*, are often used.

A popular quantization constraint is one where each BDCT coefficient is confined to a range specified by a quantization process [Zakhor, 1992][Yang et al., 1993][Weerasinghe et al., ]. A narrow quantization constraint was proposed by Park and Kim [Park and Kim, 1999]. In their method, a BDCT coefficient is limited to a range which is narrower than that defined by the popular quantization constraint. Projecting onto a narrow quantization

constraint set may result in a higher signal-to-noise ratio in a processed image when compared to projecting onto the traditional quantization constraint set. Jeong *et al.* introduced an adaptive quantization constraint [Jeong et al., 2000]. In the constraint, the range of a BDCT coefficient is determined by the local activity of the underlying block and its neighbours.

Various smoothness constraints have been proposed. Zakhor applies an FIR low-pass filter as a smoothness constraint [Zakhor, 1992]. Since it is not an ideal filter, its repeated application often makes a decoded image excessively blurry [Reeves and Eddins, 1993]. Yang *et al.* impose a smoothness constraint based on reducing the intensity variations between the boundaries of adjacent blocks [Yang et al., 1993]. After the constraint is applied, new intensity discontinuities inside each block may be generated, and hence blocking artifacts are often still obvious. Weerasinghe *et al.* construct smoothness constraint sets by segmenting a coded image into homogeneous regions [Weerasinghe et al., ]. This method is mainly effective for restoring images consisting of relatively flat areas.

There are other methods for reducing blocking artifacts. The discontinuities between blocks can be characterized by high-frequency components. Such components can be suppressed by low-pass filtering [Reeve and Lim, 1984][Liou, 1991]. However, a decoded image may be unnecessarily blurred since the high-frequency components of the original image are also suppressed. Better results can be obtained using adaptively low-pass filtering techniques. These techniques are based on the classification of blocks according to certain properties, such as edge [Ramamurthi and Gersho, 1986], texture [Meier et al., 1999], and block activity [Chen et al., 2001]. After classification, different filters are applied to different types of blocks so that artifacts are reduced while image details are preserved. Blocking artifacts can also be reduced using the wavelet transform [Choi and Kim, 2000][Wu et al., ].

POCS-based methods have a major advantage over non-POCS-based methods. In a POCS-based method, prior knowledge about an image before compression can be exploited to form constraints. These constraints make the resulting image satisfy certain requirements of the original image. However, there is no guarantee that such requirements can be met using a non-POCS-based method.

This study is aimed at reducing the blocking artifacts of BDCT coded images while avoiding excessively blurring them. A POCS-based postprocessing method is proposed. In the method, a new smoothness constraint set is suggested. The set is based on simulating an image before compression by a locally adaptive triangular mesh. To construct such a mesh, each image block is divided into a set of triangles.

In the following, a summary of the POCS theory and its application to the reduction of blocking artifacts is given in Section 2. The proposed method is described in Section 3. Experimental results and comparative studies are reported on in Section 4. The conclusion of the paper is presented in Section 5.

## 2 POCS theory and its application to reducing blocking artifacts

Let  $\{\mathbf{f}_n\}$  be a sequence in a set  $C$  such that  $\mathbf{f}_n \rightarrow \mathbf{f}^*$ .  $C$  is *closed* if  $\mathbf{f}^*$  is also in  $C$ . A set  $C$  is *convex* if

$$\alpha \mathbf{f} + (1 - \alpha) \mathbf{g} \in C \quad (1)$$

for all  $\mathbf{f}, \mathbf{g} \in C$  and  $0 \leq \alpha \leq 1$ . Given  $m$  closed convex sets  $C_i$ ,  $i = 1, 2, \dots, m$  in a Hilbert space  $\mathcal{H}$ , assume that

$$C_0 = \bigcap_{i=1}^m C_i \quad (2)$$

is nonempty. Then, for every point  $\mathbf{f}_0 \in \mathcal{H}$ , the sequence

$$\mathbf{f}_n = P_m P_{m-1} \cdots P_1 \mathbf{f}_{n-1}, \quad n = 1, 2, 3, \dots \quad (3)$$

where  $P_i$ , defined by

$$\|\mathbf{f} - P_i \mathbf{f}\| = \min_{\mathbf{g} \in C_i} \|\mathbf{f} - \mathbf{g}\|, \quad (4)$$

is the projector onto  $C_i$ , converges to a point of  $C_0$  [Youla and Webb, 1982].

An  $N \times N$  digital image  $\mathbf{f}$  can be represented by an  $N^2 \times 1$  vector

$$\mathbf{f} = [f_1, f_2, \dots, f_{N^2}]^t, \quad (5)$$

where  $t$  denotes matrix transpose, in the Euclidean space  $R^{N^2}$ , which is a Hilbert space. Then, the key issue in applying the POCS theory to removing blocking artifacts in a BDCT coded image lies in describing the known properties of the image by closed convex sets, and computing their projectors. The properties of a coded image, which can be exploited in a decoder, include the quantized BDCT coefficients, the quantization table, the prior knowledge that the original image should be block-free, and the knowledge that the intensity of each pixel should be within a limited range (e.g.,  $[0, 255]$ ). Then, a coded image  $\mathbf{f}$  can be alternately projected onto the convex sets until it converges. The converging point  $\mathbf{f}^*$  in the  $R^{N^2}$  space possesses all the known properties of the original image.

## 3 Convex sets and their projectors

Three convex sets, namely a smoothness constraint set, a quantization constraint set, and an intensity constraint set, are used in the proposed method.

### 3.1 Smoothness constraint set

A coded image is simulated by a locally adaptive triangular mesh. A block is divided into a set of triangles by choosing certain points in the block as constructing sites. The intensity of a site is determined by the intensities of its surrounding pixels while the intensities of other points in the block are approximated by the triangulation. The number of triangles in a block is determined according to the intensity variation of the block. More triangles are required to simulate a block with a large intensity variation than a block with a small intensity variation. The mesh

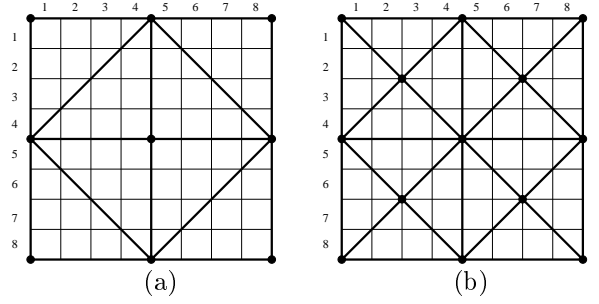


Figure 1: (a) An  $8 \times 8$  block is divided into 8 triangles if  $\sigma \leq \sigma_0$ . (b) Otherwise, it is divided into 16 triangles.

helps to reduce intensity transitions between blocks, and hence reduce the blocking effects.

It is assumed that each block consists of  $8 \times 8$  pixels in this case. The intensity variation of a block is measured by its standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{64} (I_i - \bar{I})^2}{64}} \quad (6)$$

where  $I_i$  is the intensity of pixel  $i$ , and  $\bar{I}$  is the mean intensity value of the block. A block is divided into 8 triangles if  $\sigma \leq \sigma_0$  (Figure 1(a)), where  $\sigma_0$  is a division threshold. Otherwise, it is divided into 16 triangles (Figure 1(b)). The intensity of a site (shown as a black dot in Figure 1) is the average intensity of its 4 surrounding pixels.

Let an image  $\mathbf{f}$  be divided into  $M$  triangular regions  $\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_M$ , such that

$$\mathbf{f} = \bigcup_{i=1}^M \mathbf{r}_i; \quad \mathbf{r}_i \cap \mathbf{r}_j = \phi \quad (i \neq j). \quad (7)$$

The smoothness constraint is formulated as follows:

$$C_S = \{\mathbf{f} : \|\mathbf{I}_i - \mathbf{I}_i^0\| \leq \epsilon_i, \quad i = 1, 2, \dots, M\}, \quad (8)$$

where

1.  $\mathbf{I}_i$  is a vector representing the pixel intensities of  $\mathbf{r}_i$ ,

$$\mathbf{I}_i = [I_{i1}, I_{i2}, \dots, I_{iK_i}]^t, \quad (9)$$

where  $I_{ij}$  is the intensity of a pixel at coordinates  $(x_j, y_j)$ , and  $K_i$  is the number of pixels in  $\mathbf{r}_i$ ;

2.  $\mathbf{I}_i^0$  is a constant vector,

$$\mathbf{I}_i^0 = [I_{i1}^0, I_{i2}^0, \dots, I_{iK_i}^0]^t, \quad (10)$$

where

$$I_{ij}^0 = a_i x_j + b_i y_j + c_i, \quad (11)$$

where the constants  $a_i$ ,  $b_i$  and  $c_i$  can be determined by the coordinates and intensities of the constructing sites of  $\mathbf{r}_i$ ;

3.  $\epsilon_i$  is an error bound.

It can be proven that the set  $C_S$  is convex and closed [Youla and Webb, 1982].

The projection of an image  $\mathbf{f}$  onto  $C_S$  can be obtained by computing the individual projection of each region  $\mathbf{r}_i$ . Let  $\mathbf{I}'_i$  be the projection of  $\mathbf{I}_i$  onto  $C_S$ . Then,  $\mathbf{I}'_i = P_S \mathbf{I}_i$  where  $P_S$  is the projection operator.  $\mathbf{I}'_i$  can be obtained by minimizing the following Lagrange functional:

$$J(\mathbf{I}'_i) = \|\mathbf{I}_i - \mathbf{I}'_i\|^2 + \lambda(\|\mathbf{I}_i - \mathbf{I}_i^0\|^2 - \epsilon_i^2). \quad (12)$$

Letting

$$\frac{\partial J(\mathbf{I}'_i)}{\partial I'_{ij}} = 0 \quad (13)$$

yields

$$I'_{ij} = \xi I_{ij} + (1 - \xi) I_{ij}^0 \quad (14)$$

where  $\xi = \frac{\epsilon_i}{\|\mathbf{I}_i - \mathbf{I}_i^0\|}$ .

### 3.2 Quantization constraint set

For an  $N^2 \times 1$  BDCT coded image  $\mathbf{f}$ , the quantization constraint can be expressed as follows [Yang et al., 1993]:

$$C_Q = \{\mathbf{f} : F_i^{min} \leq (T\mathbf{f})_i \leq F_i^{max}, i = 1, 2, \dots, N^2\} \quad (15)$$

where  $T$  denotes the BDCT, and  $F_i^{min}$  and  $F_i^{max}$ , determined by the quantizer, are respectively the lower and upper bounds of a BDCT coefficient before quantization.  $C_Q$  is convex and closed. The projection  $P_Q$  onto  $C_Q$  can be expressed as follows:

$$P_Q \mathbf{f} = T^{-1} \mathbf{F} \quad (16)$$

where

$$F_i = \begin{cases} F_i^{min} & \text{if } (T\mathbf{f})_i < F_i^{min}, \\ F_i^{max} & \text{if } (T\mathbf{f})_i > F_i^{max}, \\ (T\mathbf{f})_i & \text{otherwise.} \end{cases} \quad (17)$$

### 3.3 Intensity constraint set

Only grey level images are considered in this case. It is assumed that the intensity of a grey level image lies in the range  $[0, 255]$ . The intensity constraint can be described as follows:

$$C_I = \{\mathbf{f} : 0 \leq f_i \leq 255, i = 1, 2, \dots, N^2\}. \quad (18)$$

$C_I$  is convex and closed. Let  $\mathbf{f}'$  be the projection of  $\mathbf{f}$  onto  $C_I$ . Then,  $\mathbf{f}' = P_I \mathbf{f}$  where  $P_I$  is the projector, and

$$f'_i = \begin{cases} 0 & \text{if } f_i < 0, \\ 255 & \text{if } f_i > 255, \\ f_i & \text{otherwise.} \end{cases} \quad (19)$$

### 3.4 The algorithm

The proposed algorithm is expressed below:

$$\mathbf{f}_n = P_I P_Q P_S \mathbf{f}_{n-1}, n = 1, 2, 3, \dots \quad (20)$$

where  $\mathbf{f}_0$  is a BDCT coded image. The above iteration continues until the convergence is reached.

## 4 Experimental results

The proposed method has been tested on JPEG images coded at different bits per pixel (bpp). The division threshold is  $\sigma_0 = 9.0$ . The error bound of a region is  $\epsilon_i = 0.01\sqrt{K_i}$ , where  $K_i$  is the number of pixels in the region. Figure 2 shows the postprocessing result of an  $512 \times 512$  coded image (PEPPERS). The results of two competing methods, namely Zakhor's method (ZM) [Zakhor, 1992] and Yang *et al.*'s method (YGKM) [Yang et al., 1993], are included in the figure for comparison. It can be seen that the blocking artifacts in the coded image have been reduced effectively using the proposed method. On the other hand, the coded image is excessively blurred using ZM while YGKM's result still contains obvious blocking artifacts.



(a)



(b)

Figure 2: Results on PEPPERS coded at 0.218 bpp. (a) JPEG. (b) Proposed method. (to be continued.)

The objective quality of an  $N \times N$  postprocessed image  $\mathbf{f}^*$  is measured by its peak signal-to-noise ratio (PSNR), which is defined in decibels by

$$PSNR = 10 \log_{10} \left[ \frac{N^2 \times 255^2}{\|\mathbf{f}^* - \mathbf{f}\|^2} \right], \quad (21)$$

where  $\mathbf{f}$  is the original image before compression. The norm  $\|\mathbf{f}^* - \mathbf{f}\|$  is a measure of the distance between the processed image and its original. Thus, a good processed image should exhibit a high PSNR.

Figure 3 shows a summary of the PSNR at convergence on the PEPPERS and ZELDA images coded at different bit rates. It can be seen that the proposed method produces higher PSNR images when compared to ZM and YGKM.

## 5 Conclusion

Using the proposed method, the blocking artifacts of a BDCT coded image can be reduced effectively while



(c)



(d)

Fig. 2. (continued.) Results on PEPPERS coded at 0.218 bpp. (c) ZM. (d) YGKM.

the image is not excessively blurred. Subjectively, the method produces visually more appealing images than two competing methods. Objectively, it outperforms the competing methods in terms of PSNR.

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Image Method	PEPPERS (0.218 bpp)	PEPPERS (0.164 bpp)	ZELDA (0.185 bpp)	ZELDA (0.143 bpp)
JPEG	29.42	27.27	31.21	29.06
ZM	28.43	27.02	31.24	29.99
YGKM	30.05	28.14	32.09	30.16
Proposed	<b>30.28</b>	<b>28.44</b>	<b>32.35</b>	<b>30.74</b>

Figure 3: PSNR results (dB) of PEPPERS and ZELDA coded at different bit rates.

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