A Quantitative Analysis for Decomposing Visual Signal of the Gaze Displacement

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Abstract

When the head is unrestrained, the large amplitude gaze shifts are composed of coordinated eye and head movements. The position of the eyes at the onset of the gaze shift can alter the eye and head contributions to the movement. The movements of the eyes and head during the unrestrained-head gaze shift follow well-defined relationships. It is possible to predict the displacement of the eye and head components of the gaze shift, if the position of the target signal in the retina and the initial eye position in the orbit are known. In the present study we provide a quantitative analysis for decomposing the gaze displacement signal and propose a mathematical model to predict the displacement of the eye and head components of the gaze shift. We conclude that the displacements of the eye and the head components are determined by five factors: the current gaze displacement, the initial eye position, the upper limit of the range in which the eyes move freely without head contribution, the lower limit of the range in which the head moves only, and the maximal displacement of the visual target (maximal initial retinal error). The results of our proposed model are inline with data observed physiologically.

Keywords: Gaze shift, Eye and head movements, Visual signal.

1 Introduction

Behavioural studies using monkeys and humans as subjects show that the gaze shift are accomplished by moving both the eyes and the head in the same direction toward the target when the head is unrestrained (Bizzi et al. 1972; Barnes 1979; Guitton and Volle, 1987; Monoz et al. 1991; Sparks and Groh, 1995; Misslisch et al. 1998). The movements of the eyes and head during the unrestrained-head gaze shift follow a tight linear relationship linking head contribution and gaze amplitude. In general, these results are similar to those reported previously for unrestrained-head gaze shifts in the primate, directed along the horizontal meridian (Bizzi et al. 1972; Phillips et al. 1995; Tomlinson 1990; Gresty 1974; Guitton and Volle 1987; Tweed et al. 1995; Zangmeister and Stark 1982). The head and eye movement amplitudes are influenced by the initial position of the eyes in orbit (Becker, 1992). Freedman and Sparks (1997) have reported experimental data concluding that it is possible to accurately predict the amplitudes of the eye and head components of the gaze shift if the displacement of the target and the initial position of the eyes in the orbits are known. The prediction model we describe here is based on this experimental data.

For simplicity, the following discussion is restricted to horizontal gaze shift, and neglects the other two rotational degrees of freedom of the eyes. It is well known that the gaze shift is the sum of the eye-in-head movement and head-in-space movement (Bizzi et al. 1972). Considering the effect of the initial position of an eye in the orbit, we have

$$G = E(p) + H(p) \qquad (1)$$

where G is the gaze displacement; p, the initial position of the eye in the orbit; E, the eye displacement; H, the head movement contribution. The head contribution to the gaze shift is defined as the amplitude of the head movement that occurred between head movement onset and gaze movement-end (Freedman and Sparks, 1997).

The purpose of the present model is to predict the eye displacement $E(p)$ and the head movement contribution $H(p)$ if the gaze displacement G and the initial position p of the eyes in the orbits are known.

2 Models

2.1 Horizontal gaze shift with eyes centred

When the eyes are initially centered in orbit, the relationship between the eye amplitude and the gaze amplitude is characterized by two linear functions, shown in Figure 1, (Freedman and Sparks, 1997).

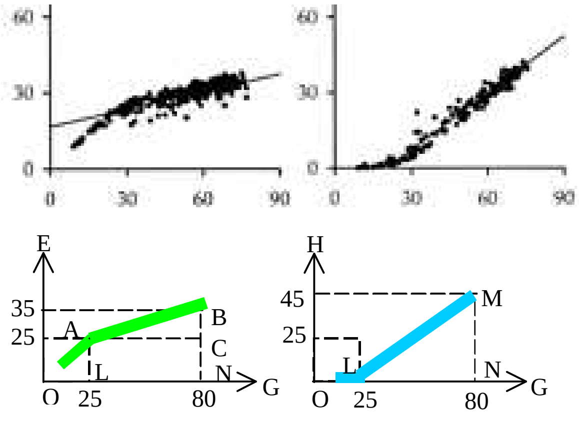

Figure 1. Left, a piecewise linear relationship between the eye amplitude and the gaze amplitude, and right, a piecewise linear relationship between the head contribution and gaze amplitude, when the eyes are initially centered in orbits. E, eye amplitude; H, head contribution; G, gaze amplitude. The top graphs are derived from Figure 14, Freedman and Sparks (1997).

When the gaze amplitude $G \le 25°$, the eyes move without head contributions. The eye amplitude increases equally with increasing gaze amplitude, and the slope of the linear relationship between the eye and gaze amplitudes is equal to 1.

$$E = G, \quad and \quad k_E = \frac{E}{G} = 1 \qquad When \quad G \le 25 \tag{2}$$

where E is the eye amplitude; G, the gaze amplitude; k_E, the slope of the linear relationship between eye and gaze amplitudes.

The head movement does not contribute to change in gaze position, and the slope of the linear relationship (ratio) between head contribution and gaze amplitude equal to 0.

When the gaze amplitude $G > 25°$, the head begins to contribute to the gaze shift. Assuming a subject has a maximal visual target offset of 80° (maximal initial retinal error), the head contribution increases linearly with increasing gaze amplitude for movements between 25° and 80°. Because the horizontal gaze amplitude is approximately equal to the target displacement amplitude (Freedman and Sparks, 1997), we describe here the maximal gaze amplitude using maximal offset of visual target. The maximal gaze amplitude involves only the eyes and head movements.

When the gaze amplitude D reaches its maximum 80°, the eye movement amplitude saturates at amplitude 35°, shown in the left graph in Figure 1 (Freedman and Sparks, 1997). When the gaze amplitude is between 25° and 80°, K_E is

$$\begin{aligned} k_E &= \frac{E}{G} = \frac{eye \quad amplitude}{gaze \quad amplitude} = \frac{BN - CN}{ON - OL} = \frac{BC}{AC} \\ &= \frac{35 - 25}{80 - 25} = \frac{10}{55} = 0.18 \qquad when \quad 25 < G < 80 \end{aligned} \tag{3}$$

where BN = 35°, the maximal amplitude of the eye; ON = 80°, the maximal displacement of the gaze; CN = AL = OL = 25°, the position from which the head begins to contribute to the gaze shift.

When the gaze amplitude G is equal to 80°, the head reaches its maximal position. The maximal contribution of the head is the difference between the maximal amplitude of the gaze and the maximal amplitude of the eye, and is equal to 80-35=45, shown in the right graph in Figure 1. The slope, K_H, of the linear ratio of the head contribution and the gaze amplitude is

$$\begin{aligned} k_H &= \frac{H}{G} = \frac{the\ maximum\ of\ head\ contribution}{gaze\ amplitude} \\ &= \frac{MN}{LN} = \frac{80-35}{80-25} = \frac{45}{55} = 0.82 \qquad when\ 25 < G < 80 \end{aligned} \tag{4}$$

where MN = (the maximal amplitude of the gaze) – (the maximal amplitude of the eye); LN = (the maximal amplitude of the gaze) – (the position from which the head begins to contribute to the gaze shift).

As the gaze amplitude is the sum of the eye amplitude and the head contribution, it is clear that

$$k_E + k_H = \frac{E}{G} + \frac{H}{G} = \frac{G}{G} = 1 \tag{5}$$

When the eyes are initially centered in orbit, the gaze shift can be described as

$$\begin{aligned} G &= E(0) + H(0) \\ &= \begin{cases} E & when \quad G \le 25 \\ 25 + k_E(G - 25) + k_H(G - 25) & when \quad 25 < G < 80 \end{cases} \end{aligned} \tag{6}$$

The eye amplitude and the head contribution can be respectively described as

$$\begin{aligned} E(0) &= \begin{cases} G & when \quad G \le 25 \\ 25 + k_E(0) \cdot (G - 25) & when \quad 25 < G < 80 \end{cases} \\ H(0) &= \begin{cases} 0 & when \quad G \le 25 \\ k_H(0) \cdot (G - 25) & when \quad 25 < G < 80 \end{cases} \end{aligned} \tag{7}$$

2.2 Horizontal gaze shift with contralateral position of the eye

When the initial position of the eyes in the orbits is at 10° contralateral to the direction of the gaze shift, the relationship between eye amplitude and gaze amplitude is characterized by two linear functions, shown in Figure 2 (Freedman and Sparks, 1997).

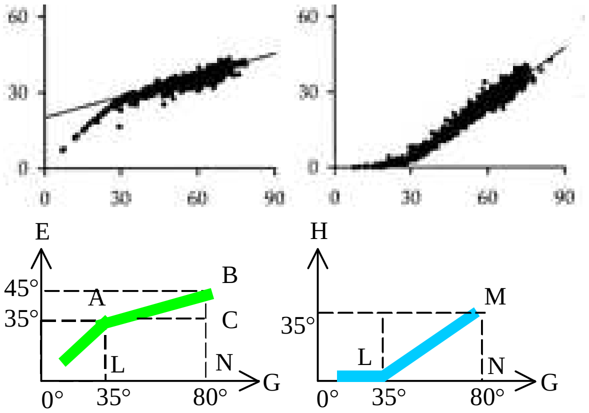

Figure 2. Left, a piecewise linear relationship between the eye amplitude and the gaze amplitude, and right, a piecewise linear relationship between the head contribution and gaze amplitude, when the initial position of the eye in the orbits is at 10° contralateral to the direction of the gaze shift. E, eye amplitude; H, head contribution; G, gaze amplitude. The top graphs are derived from Figure 14, Freedman and Sparks (1997).

When the gaze amplitude $G \le 35°$, the eyes move without head contributions. The eye amplitude is equal to gaze amplitude, and the slope of the linear relationship between eye amplitude and gaze amplitude is equal to 1.

$$E = G, \quad and \quad k_E = \frac{E}{G} = 1 \qquad When \quad G \le 35 \tag{8}$$

The head movement does not contribute to changes in gaze position, and the slope of the linear relationship between head contribution and gaze amplitude is equal to 0.

When the gaze amplitude $G > 35°$, the head begins to contribute to the gaze shift, assuming a subject has a maximal gaze displacement 80°.

When the gaze amplitude G = 80°, the eye movement amplitude saturates at 45°. Because the range of the gaze amplitude is from 35° to 80°, the slope of the linear relationship between eye amplitude and gaze amplitude is

$$k_E(10)=\frac{E}{G}=\frac{\text{eye amplitude}}{\text{gaze amplitude}}=\frac{BN-CN}{ON-OL}=\frac{BC}{AC} \tag{9}$$
$$=\frac{45-35}{80-35}=\frac{10}{45}=0.22 \qquad \text{when } 35<G<80$$

When the gaze amplitude G = 80°, the head reaches its maximal position. The maximal contribution of the head is equal to 80-45=35, shown in the right graph of Figure 2. The slope of the linear relationship between head contribution and gaze amplitude is

$$k_H(10)=\frac{H}{G}=\frac{\text{the maximum of head contribution}}{\text{gaze amplitude}}=\frac{MN}{LN} \tag{10}$$
$$=\frac{80-45}{80-25}=\frac{35}{55}=0.78 \qquad \text{when } 35<G<80$$

When the initial eyes position in the orbits is at 10° contralateral to the direction of the gaze shift, the gaze shift can be described as

$$G=E(10)+H(10)=\begin{cases} E & \text{when } G\le 35 \\ 35+k_E(G-35)+k_H(G-35) & \text{when } 35<G<80 \end{cases} \tag{11}$$

The eye amplitude and the head contribution can be described as

$$E(10)=\begin{cases} G & \text{when } G\le 35 \\ 35+k_E(10)\cdot(G-35) & \text{when } 35<G<80 \end{cases} \tag{12}$$
$$H(10)=\begin{cases} 0 & \text{when } G\le 35 \\ k_H(10)\cdot(G-35) & \text{when } 35<G<80 \end{cases}$$

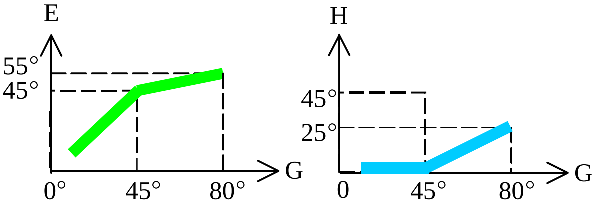

Figure 1. K_E (left) and K_H (right) when the initial position of the eye in the orbits is at 20° contralateral to the direction of the gaze shift.

Similarly, Equations 13 and 14 describe the slope k_E and k_H when the eyes initial deviation in orbit are 20° and 30° contralateral to the direction of the gaze shift respectively, shown in Figure 3 and Figure 4.

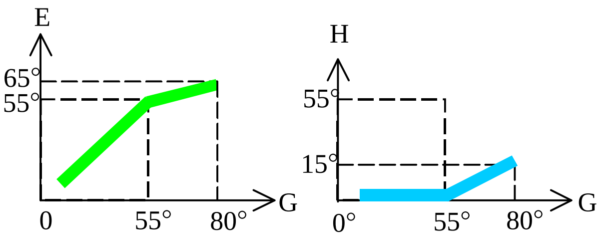

Figure 4. K_E (left) and K_H (right) when the initial position of the eye in the orbits is at 30° contralateral to the direction of the gaze shift.

$$k_E(20)=\frac{(35+20)-45}{80-45}=\frac{10}{35}=0.29 \tag{13}$$
$$k_H(20)=\frac{(80-45)-10}{80-45}=\frac{25}{35}=0.71$$

$$k_E(30)=\frac{(35+30)-55}{80-55}=\frac{10}{25}=0.4 \tag{14}$$
$$k_H(30)=\frac{55-30-10}{55-30}=\frac{15}{25}=0.6$$

The results of the equations are close to the observed physiological data (Freedman and Sparks, 1997), shown in Table 1 and Figure 5.

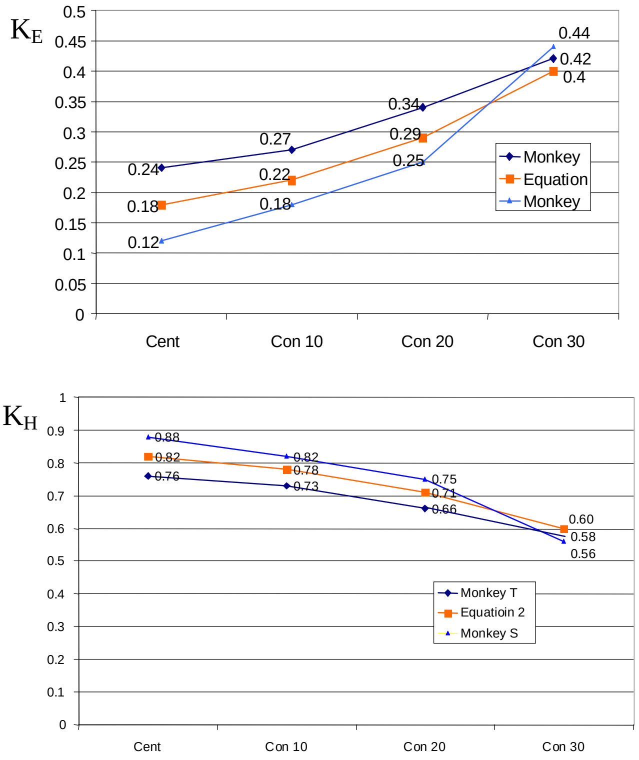

Figure 5. Compare the results of the model with the physiological data. The dark line and blue line are the data reported by Freedman and Sparks (1997). The red thick line is the result of our Equations. "Cent", the eyes are initially centred in the orbit. "Con", the eye position in the orbit is contralateral to the direction of the gaze shift.

	Eye amplitude				Head contribution			
	0	10	20	30	0	10	20	30
T	0.24	0.27	0.34	0.42	0.76	0.73	0.66	0.58
Eq.	0.18	0.22	0.29	0.40	0.82	0.78	0.71	0.60
S	0.12	0.18	0.25	0.44	0.88	0.82	0.75	0.56

Table 1 Comparison between results of the model and physiological data. T and S correspond to data from monkeys T and S (Freedman and Sparks, 1997), and Eq. corresponds to the results of our proposed model.

The data in Table 1 are the slopes of the linear relationship between eye amplitude and gaze amplitude (left column), and between head contribution and gaze amplitude (right column). The numbers 10, 20, 30, correspond to the initial eye position in orbit of 10°, 20°, 30° contralateral to the direction of the gaze shift. Note that the data of the head contribution of the monkey S in the last column in the Table 2 reported by Freedman and Sparks (1997), may have a typographical error (it is shown as 0.56 but might be 0.44 as the sum of k_H and k_E is equal to 1).

If the range of the initial position of the eyes in orbit is from 0° to 30° contralateral to the direction of the gaze shift, we have:

$$k_E(p)=\begin{cases} 1 & \text{when } G \le 25+p \\ 10/(55-p) & \text{when } G > 25+p \end{cases} \tag{15}$$

$$\begin{aligned} k_H(p) &= 1-k_E(p) \\ &= \begin{cases} 1 & \text{when } G \le 25+p \\ (45-p)/(55-p) & \text{when } G > 25+p \end{cases} \end{aligned} \tag{16}$$

where p is the initial eye position contralaterally to the direction of the gaze shift.

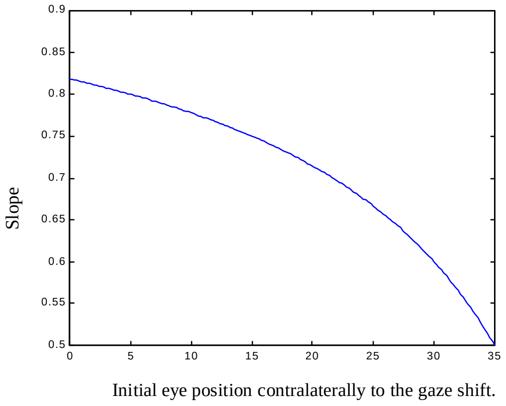

Figure 6. The slope of the linear relationship between head contribution and gaze amplitude decreases as the eyes initial position in orbit is at increasing contralateral positions.

The curve of Equation 16 is shown in Figure 6. The slope of the linear relationship between head contribution and gaze amplitude decreases as the eyes initial positions in orbit are at increasing contralateral positions. This result is similar to the data reported by Freedman and Sparks (1997).

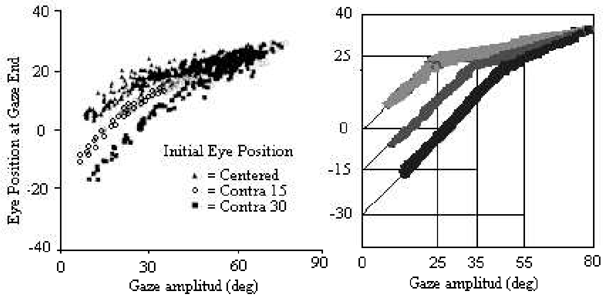

Figure 7. Eye position at the end of the gaze shift plotted as a function of gaze amplitude for movements initiated with the eyes in different initial positions. In the left graph, data reported by Freedman and Sparks (1997). In the right graph, data from our model.

The eye amplitude and the head contribution can be respectively described by the Eye Amplitude Equation and the Head Contribution Equation:

$$\begin{aligned} E(G,p) &= \begin{cases} G & \text{when } G \le 25+p \\ 25+p+k_E(p)\cdot(G-(25+p)) & \text{when } G > 25+p \end{cases} \\ &= \begin{cases} G & \text{when } G \le 25+p \\ 25+p+\dfrac{10}{55-p}\cdot(G-(25+p)) & \text{when } G > 25+p \end{cases} \end{aligned} \tag{17}$$

$$\begin{aligned} H(G,p) &= \begin{cases} 0 & \text{when } G \le 25+p \\ k_H(p)\cdot(G-(25+p)) & \text{when } G > 25+p \end{cases} \\ &= \begin{cases} 0 & \text{when } G \le 25+p \\ \dfrac{45-p}{55-p}\cdot(G-(25+p)) & \text{when } G > 25+p \end{cases} \end{aligned} \tag{18}$$

Equation 17 and 18 predict the eye amplitude E and the head contribution H if the gaze displacement G and the initial eye position p are known.

The gaze amplitude is the sum of eye amplitude and head contribution. We have the Gaze Decomposition Equation:

$$\begin{aligned} G &= E(p)+H(p) \\ &= \begin{cases} E & \text{when } G \le 25+p \\ 25+p+k_E(G-(25+p))+k_H(G-(25+p)) & \text{when } G > 25+p \end{cases} \end{aligned} \tag{19}$$

If the initial position of the eyes and the eye amplitude are known, the eye position at the end of the gaze shift is their algebraic sum:

$$P_E = E+(-p) = E-p \tag{20}$$

From Equation 20, the eye position at the end of the gaze shift is described by the Eye Position Equation:

$$\begin{aligned} P_E &= \begin{cases} G-p & \text{when } G \le 25+p \\ 25+k_E\cdot(G-(25+p)) & \text{when } G > 25+p \end{cases} \\ &= \begin{cases} G-p & \text{when } G \le 25+p \\ 25+\dfrac{10}{55-p}\cdot(G-(25+p)) & \text{when } G > 25+p \end{cases} \end{aligned} \tag{21}$$

where P_E is the eye position at the end of the gaze shift; p, the initial position of the eyes; G, the gaze amplitude. The position of the eyes at the end of the gaze shift necessarily depends on both the initial eye position in orbit and the amplitude of the gaze shift. As Figure 7 shows, the results produced by our model (Equation 21) are qualitatively and quantitatively very close to the recorded physiological data of Freedman and Sparks (1997).

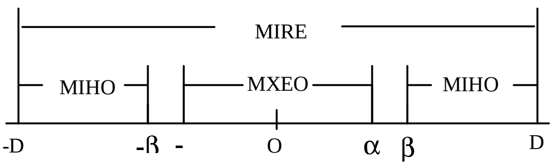

Figure 8. MEXO, range of the maximum eye-movement-only; α, upper limit of MEXO (typically between 25° and 35° for monkey). MIHO, range of the head-movement-only; β, the lower limit of the range (typically between 35° and 45° for monkey). The head moves only if the eyes initial positions in the orbits are at β ipsilateral to the direction of the gaze shift. MIRE, range of the maximal initial retinal error. D, the maximal initial retinal error (typically between 80° and 90°). O, the center of the retina.

2.3 Factors determining gaze decomposition

Besides the current gaze displacement G and the initial eye position p there are other three factors have to be considered in gaze decomposition in the gaze decomposition model:

MXEO and α. For the horizontal gaze shifts with eyes centered and gaze shifts smaller than about 25°, experimental data indicate that the head movement does not contribute to the change in gaze position (Sparks and Groh, 1995: Tweed et al. 1995; Freedman and Sparks, 1997). The range between -25° and 25° is called the maximum eye-movement-only (MXEO), shown in Figure 8, because within this range the eyes move freely without head contributions. The upper limit of the range is noted α, which value is typically between 25° and 35° for monkey.

MIHO and β. For gaze shifts larger than about 35°, eye position at the end of the gaze shift does not exceed 35°, and the maximal contribution of the eyes is below 45° (Guitton and Volle, 1987). The range that is greater than 35° is called the minimum head-movement-only (MIHO), because the head moves only if the eyes initial positions in the orbits are at 35° ipsilateral to the direction of the gaze shift (Becker, 1992). The lower limit of the range is noted β. The value of β is typically between 35° and 45° for monkey.

MIRE and D. When a visual target is beyond about 80° from the center of the retina, we cannot see it. The range within 80° is called the maximal initial retinal error (MIRE) or maximal gaze amplitude. The maximal initial retinal error is noted D, which value is typically between 80° and 90°.

The value of α, β, and D are fixed for a subject even though they may have ±5° errors. However, different subjects may have different α, β, and D, depending on their habits, the size of their eye-orbits, and their vision capabilities.

2.4 A summary of the Equations

Slope Equation

$$k_E(p) = \begin{cases} 1 & \text{when } G \le \alpha + p \\ \dfrac{\beta - \alpha}{D - p - \alpha} & \text{when } G > \alpha + p \end{cases} \qquad (22)$$

$$k_H(p) = \begin{cases} 1 & \text{when } G \le \alpha + p \\ \dfrac{D - p - \beta}{D - p - \alpha} & \text{when } G > \alpha + p \end{cases} \qquad (23)$$

Eye Amplitude Equation

$$\begin{aligned} E(G,p) &= \begin{cases} G & \text{when } G \le \alpha + p \\ \alpha + p + k_E(p) \cdot (G - (\alpha + p)) & \text{when } G > \alpha + p \end{cases} \\ &= \begin{cases} G & \text{when } G \le \alpha + p \\ \alpha + p + \dfrac{\beta - \alpha}{D - p - \alpha} \cdot (G - (\alpha + p)) & \text{when } G > \alpha + p \end{cases} \end{aligned} \qquad (24)$$

Head Contribution Equation

$$\begin{aligned} H(G,p) &= \begin{cases} 0 & \text{when } G \le \alpha + p \\ k_H(p) \cdot (G - (\alpha + p)) & \text{when } G > \alpha + p \end{cases} \\ &= \begin{cases} 0 & \text{when } G \le \alpha + p \\ \dfrac{D - p - \beta}{D - p - \alpha} \cdot (G - (\alpha + p)) & \text{when } G > \alpha + p \end{cases} \end{aligned} \qquad (25)$$

Gaze Decomposition Equation

$$\begin{aligned} G &= E(G,p) + H(G,p) \\ &= \begin{cases} E & \text{when } G \le \alpha + p \\ 25 + p + k_E(G - (\alpha + p)) + k_H(G - (\alpha + p)) & \text{when } G > \alpha + p \end{cases} \end{aligned} \qquad (26)$$

Eye Position Equation

$$\begin{aligned} P_E &= \begin{cases} G - p & \text{when } G \le \alpha + p \\ \alpha + k_E \cdot (G - (\alpha + p)) & \text{when } G > \alpha + p \end{cases} \\ &= \begin{cases} G - p & \text{when } G \le \alpha + p \\ \alpha + \dfrac{\beta - \alpha}{D - p - \alpha} \cdot (G - (25 + p)) & \text{when } G > \alpha + p \end{cases} \end{aligned} \qquad (27)$$

Note the head contribution is only an epiphenomenon of eye amplitude, total head amplitude, and the relative timing of the two platforms. The neural system drives the total head movement and may not deal with specific head contribution (Freedman and Sparks, 1997).

2.5 A Graphic method for gaze decomposition

We explain our graphic method using Figure 9. In this figure, the length D is the maximal displacement of the gaze shift, and determines the outer square OSTW. The diagonal line OT and the side OS are known from the OSTW square. From p, α, and β, we can obtain the line PQ. For any gaze displacement G, the fourth line AB is defined, and the length of AB is G. If $G > p + \alpha$, the intersection M of AB and PQ divides AB into two segments AM and MB. From Equation 24, AM = H, the head contribution; MB = E, the eye amplitude. If $G \le p + \alpha$, then AM = 0, MB = G. This means the eyes move without head contribution.

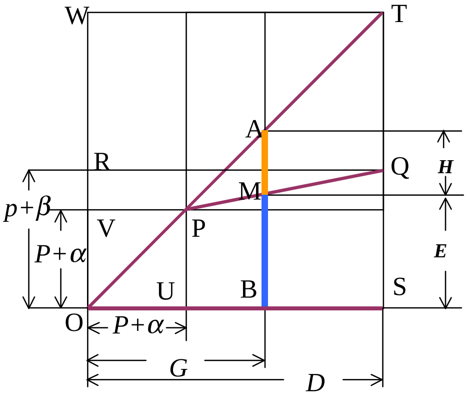

Figure 9. Graphic method for the gaze-decomposition. AM = H and MB = E are the results of the gaze-decomposition. In the square OSTW, OU = OV = $p+\alpha$, where p, the initial eye position; α, the upper limit of the range in which the eyes move freely without head contribution. OR = $p+\beta$, where β, the lower limit of the range in which the head moves only; OS = D, the maximal initial retinal error. Three lines OT, PQ and OS are determined by α, β, p, and D. For any OB = G, the gaze displacement, the line AB is crossed by PQ at M. Then AM = H, the head contribution, and MB = E, the eye amplitude.

3 Discussion

3.1 Comparison with other models

A number of models have been proposed to describe the relationship between eye and gaze amplitude of head-free gaze shifts. However, most of these models have only considered a qualitative liner relationship. Our proposed model provides a quantitative analysis for the gaze decomposition.

The results of the quantitative analysis indicate that the distribution of the eye amplitude and head contribution is determined not only by the current gaze displacement and the initial eye position, but also by the maximum eye-movement-only (MXEO), the minimum head-movement-only (MIHO) and the maximal initial retinal error (MIRE). The minimum head-movement-only and the effective oculomotor range have been well studied. Our model emphasizes that the MXEO and the MIRE play essential role for the gaze decomposition in head-free gaze shift.

3.2 Fitting errors between the model and physiological data

To assess the accuracy of our model we have studied the fitting error with respect to the physiological data (see Figure 5 and Table 1, Freedman and Sparks, 1997). When we set $\alpha = 25°$, $\beta = 35°$, and $D = 80°$, the absolute error between the result of Equation 22 and the average value of monkey T and S for k_E are: (0.24+0.18)/2 –0.18=0 (Cent), (0.27+0.18)/2–0.22=0.005 (Con 10°), (0.34+0.25) /2–0.29=0.005 (Con 20°), (0.42+0.44)/2 –0.4=0.03 (Con 30°).

If we compare the above results of the model with data of individual monkey, for example monkey T, there is rather large fitting error: 0.24-0.18=0.06 (Cent), 0.27-0.22=0.05 (Con10°), 0.34-0.29=0.05 (Con 20°), 0.42-0.40=0.02 (Con 30°). However, different monkeys would have different α, β, and D. If we set $\alpha = 23°$, $\beta = 37°$ and $D = 85°$ for monkey T in Equation 22, the fitting error will be: 0.24-0.23=0.01, 0.27-0.274=-0.004, 0.34-0.341=-0.001, 0.42 -0.45=-0.03. The relative error is less than 7%.

3.3 Prediction of the characteristic of individual monkey

Another new feature of the model is that it provides a method to predict the visual characteristic of individual monkeys such as: the upper limit of the range in which the eyes move freely without head contribution; the lower limit of the range in which the head moves only; and the maximal initial retinal error, if the current gaze displacement, the initial eye position and the results of the gaze decomposition are known. For example, from the data reported in table 2 by Freedman and Sparks (1997), we can deduce the coefficient α=23°, β=37°, D=85° for monkey T, and α=39°, β=45°, D=83° for monkey S using Equation 22 and Equation 23 in our model. As the range of eyes-movement-only of monkey S is greater than that of monkey T, we may predict that the monkey S prefer to move its eyes rather than its head for a visual target. For example, for a 30° horizontal visual target when the eyes are initially centered in orbits, Monkey S only moves its eyes to see it, but Monkey T has to move both eyes and head to see it.

3.4 Why there is liner relationship between eye amplitude and gaze displacement?

Horizontal eye position is mainly controlled by two extraocular muscles, the lateral and medial recti. When the eye is still in the range of MXEO, it is at the mechanical equilibrium point at which the net torque due to the muscles and passive orbital tissues vanishes. When eyes move within the range between MXEO and MIHO, the eyes are still in an increasingly ipsilateral position to the direction of the gaze shift, and the strength of the muscles increases. If the strength of the muscles is directly proportional to its length, the property of the muscle may lead to the linear relationship between eye amplitude and gaze displacement. Another reason for the linear relationship may be that the saturation of the eye position prevents the eye from running at full speed to the end of its leash, a manoeuvre that might overstretch the muscles or damage the globe. Further, saturation prevents errors that might arise if the muscles were given commands they could not execute.

4 Conclusion

In the present study we have presented a quantitative model for gaze decomposition in a head-free gaze shift. The model has the potential of dealing with the gaze decomposition under other conditions, for example, the initial eye position ipsilaterally to the direction of the gaze shift, or when the direction of the gaze shift is extended to the vertical or oblique direction. The results of the quantitative analysis may benefit the study of eye velocities and head movement components, for example, the velocity of the eye may be considered as the function of the slope of the linear relationship between eye amplitude and gaze amplitude.

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